

Amending Contracts for Choreographies

Laura Bocchi

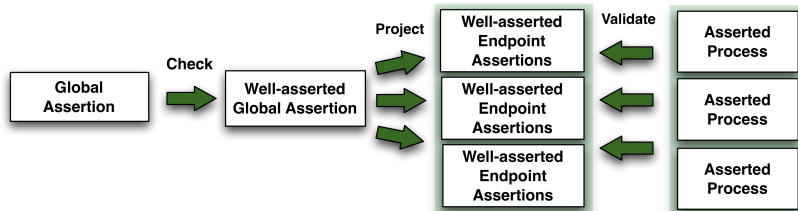
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Introduction



- ▶ Well-Assertedness
- ▶ Difficult to attain by hand
- ▶ Three algorithms:
 - ▶ Strengthening
 - ▶ Propagation
 - ▶ Lifting

cf. Honda et al. in POPL 2008
Bocchi et al. in CONCUR 2010

Background - Global Assertion

An example of global assertion:

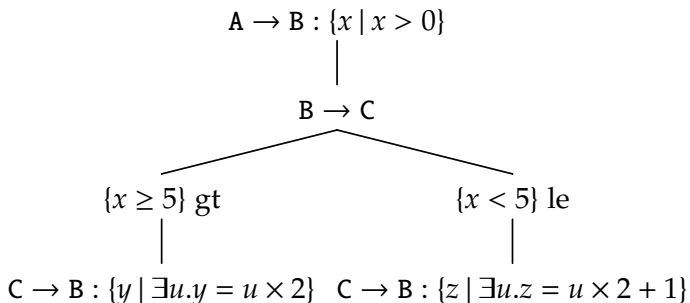
$$A \rightarrow B : \{x \mid x > 0\}.$$

$$B \rightarrow C : \begin{array}{l} \{x \geq 5\} \text{ gt} : C \rightarrow B : \{y \mid \exists u. y = u \times 2\}, \\ \{x < 5\} \text{ le} : C \rightarrow B : \{z \mid \exists u. z = u \times 2 + 1\} \end{array}$$

- ▶ Logic needs to be decidable
- ▶ Global assertions have to be closed
- ▶ ... and they must obey two conditions

Background - Global Assertion

An example of global assertion:



- ▶ Logic needs to be decidable
- ▶ Global assertions have to be closed
- ▶ ... and they must obey two conditions

Background - Well-Assertedness

- ▶ **History Sensitivity (HS):** a participant having an obligation has enough information for choosing a set of values that guarantees it,

Alice \rightarrow Bob : $\{x \mid \text{true}\}$.

Bob \rightarrow Carol : $\{y \mid \text{true}\}$.

Carol \rightarrow Alice : $\{z \mid z > x\}$

Background - Well-Assertedness

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Alice \rightarrow Bob : $\{x \mid \text{true}\}$.

Bob \rightarrow Carol : $\{y \mid \text{true}\}$.

Carol \rightarrow Alice : $\{z \mid z > x\}$

- ▶ **Temporal Satisfiability (TS):** the values sent in each interaction do not make predicates of future interactions unsatisfiable,

Alice \rightarrow Bob : $\{x \mid x > 10\}$.

Bob \rightarrow Alice : $\{y \mid y < x \wedge y > 10\}$

Outline

Introduction

Recovering History Sensitivity

Strengthening
Propagation

Recovering Temporal Satisfiability

Lifting

A Methodology

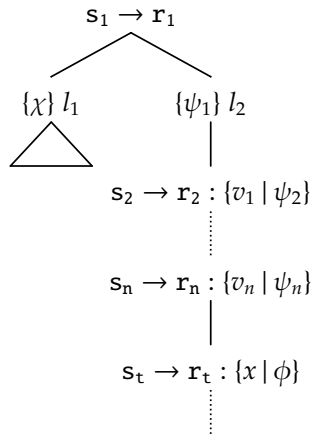
Conclusions

History Sensitivity

A participant having an obligation has enough information for choosing a set of values that guarantees it.

- ▶ A participant *knows* a variable if s/he either sends or receives it,
- ▶ a sender *must know* all the variables which appear in a predicate s/he guarantees,
- ▶ otherwise, **history sensitivity** is violated!

Strengthening



- ▶ Assume no HS problems in ψ_s ,
- ▶ s_t does not know v (appearing in ϕ).
- ▶ Find a variable v_i :
 - ▶ v_i is known to s_t
 - ▶ $(\psi_1 \wedge \psi_2 \wedge \dots \wedge \psi_n) \wedge \phi[v_i/v] \Rightarrow \phi$ holds?
- ▶ Replace ϕ by $\phi[v_i/v]$

Strengthening: Example

$A \rightarrow B : \{v_0 \mid v_0 \leq 50\}.$

$B \rightarrow C : \{v_1 \mid v_1 > v_0\}.$

$C \rightarrow D : \{v_2 \ v_3 \mid v_2 > v_1 \wedge v_3 \leq 70\}.$

$C \rightarrow D : \{v_1 \geq 0\}$ **pos** : $D \rightarrow C : \{v_4 \mid v_4 > v_0\},$
 $\{v_1 < 0\}$ **neg** : $D \rightarrow C : \{v_5 \mid v_5 < v_0\}$

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Can we find v_i such that D knows v_i and

$$\underbrace{(v_0 \leq 50 \wedge v_1 > v_0 \wedge v_2 > v_1 \wedge v_3 \leq 70)}_{\psi_1 \wedge \psi_2 \wedge \dots \wedge \psi_n} \wedge \underbrace{(v_4 > v_i)}_{\phi[v_i/v]} \Rightarrow \underbrace{(v_4 > v_0)}_{\phi}$$

holds?

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holds? Yes, D knows v_2 for which the equation above holds!

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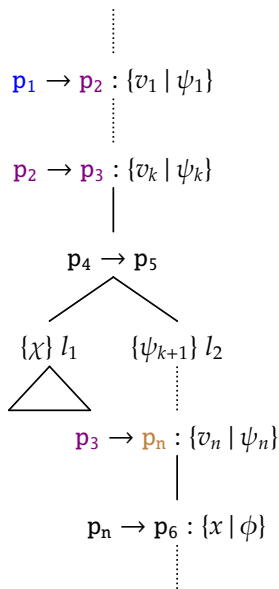
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holds? No, D doesn't know any variable for which the equation above holds...

Propagation



- ▶ Assume no HS problems in ψ s,
- ▶ p_n does not know v (appearing in ϕ),
- ▶ p_1 knows v .
- ▶ Find a chain of interactions:
 - ▶ the sender of the first interaction knows v ,
 - ▶ the receiver of the last interaction is p_n ,
 - ▶ the sender of a node is the receiver of the previous node in the chain.

Propagation: Example

$$A \rightarrow B : \{v_0 \mid v_0 \leq 50\}. \quad (1)$$

$$B \rightarrow C : \{v_1 \mid v_1 > v_0\}. \quad (2)$$

$$C \rightarrow D : \{v_2 \ v_3 \mid v_2 > v_1 \wedge v_3 \leq 50\}. \quad (3)$$

$$C \rightarrow D : \quad \{v_1 \geq 0\} \text{ pos} : \quad D \rightarrow C : \{v_4 \mid v_4 > v_2\}, \quad (4)$$

$$\quad \quad \quad \{v_1 < 0\} \text{ neg} : \quad D \rightarrow C : \{v_5 \mid v_5 < v_0\} \quad (5)$$

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Find a chain of *interactions* such that:

- ▶ the sender of the first interaction knows v_0 , e.g. (1)
- ▶ the receiver of the last interaction is D, e.g. (3), and
- ▶ the sender for each interaction is the receiver of the previous interaction in the chain, e.g. (1) \rightarrow (2) \rightarrow (3)

Propagation: Example

$$A \rightarrow B : \{v_0 \mid v_0 \leq 50\}. \quad (1)$$

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In the chain (1) \rightarrow (2) \rightarrow (3):

Propagation: Example

$$A \rightarrow B : \{v_0 \ u_1 \mid v_0 \leq 50 \wedge u_1 = v_0\}. \quad (1)$$

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In the chain (1) \rightarrow (2) \rightarrow (3):

- ▶ $u_1 = v_0$ is added to the predicate of the first interaction,

Propagation: Example

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In the chain (1) \rightarrow (2) \rightarrow (3):

- ▶ $u_1 = v_0$ is added to the predicate of the first interaction,
- ▶ $u_i = u_{i-1}$ is added the i^{th} interaction's predicate ($1 < i \leq 3$),

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In the chain (1) \rightarrow (2) \rightarrow (3):

- ▶ $u_1 = v_0$ is added to the predicate of the first interaction,
- ▶ $u_i = u_{i-1}$ is added the i^{th} interaction's predicate ($1 < i \leq 3$),
- ▶ u_3 replaces v_0 in the “problematic” interaction's predicate

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Temporal Satisfiability

The values sent in each interaction do not make predicates of future interactions unsatisfiable.

- ▶ All the values satisfying the predicates before an interaction must allow to instantiate its interaction variables.
- ▶ At least one branch can be chosen, and each branch must satisfy temporal satisfiability.

Lifting: Example

$$A \rightarrow B : \{x \mid x < 10\}. \quad (1)$$

$$A \rightarrow B : \{y \mid y > 8\}. \quad (2)$$

$$B \rightarrow A : \{z \mid x > z \wedge z > 6 \wedge y \neq z\} \quad (3)$$

Lifting: Example

$$A \rightarrow B : \{x \mid x < 10\}. \quad (1)$$

$$A \rightarrow B : \{y \mid y > 8\}. \quad (2)$$

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- ▶ Identify which part of the predicate is *in conflict* with the previous predicates ...

Lifting: Conflict

- ▶ Assume that z is introduced at the node where TS is violated, and ϕ is its predicate.
- ▶ Search for β such that
 - ▶ $\phi \iff \gamma \wedge \beta$, and
 - ▶ β is in conflict on z with γ in $\psi_1 \wedge \dots \wedge \psi_k$.

Conflict

The predicate β is in conflict on z with γ in $\psi_1 \wedge \dots \wedge \psi_k$ iff

$$\psi_1 \wedge \dots \wedge \psi_k \Rightarrow \exists z. \gamma \quad \text{and} \quad \psi_1 \wedge \dots \wedge \psi_k \not\Rightarrow \exists z. (\gamma \wedge \beta)$$

Lifting: Example

$$A \rightarrow B : \{x \mid x < 10\}. \quad (1)$$

$$A \rightarrow B : \{y \mid y > 8\}. \quad (2)$$

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- ▶ In our case we have that

$$\underbrace{(x < 10 \wedge y > 8)}_{\psi_1 \wedge \dots \wedge \psi_k} \Rightarrow \underbrace{\exists z. y \neq z}_{\gamma} \text{ and}$$

$$\underbrace{(x < 10 \wedge y > 8)}_{\psi_1 \wedge \dots \wedge \psi_k} \not\Rightarrow \exists z. \underbrace{x > z \wedge z > 6}_{\gamma \wedge \beta}$$

- ▶ Thus, $x > z \wedge z > 6$ is *in conflict*, and we have to “lift” it ...

Lifting: Example

$$A \rightarrow B : \{x \mid x < 10\}. \quad (1)$$

$$A \rightarrow B : \{y \mid y > 8\}. \quad (2)$$

$$B \rightarrow A : \{z \mid x > z \wedge z > 6 \wedge y \neq z\} \quad (3)$$

- ▶ “Lift” the predicate $z > 6 \wedge x > z$:

Lifting: Example

$$A \rightarrow B : \{x \mid x < 10\}. \quad (1)$$

$$A \rightarrow B : \{y \mid y > 8\}. \quad (2)$$

$$B \rightarrow A : \{z \mid x > z \wedge z > 6 \wedge y \neq z\} \quad (3)$$

- ▶ “Lift” the predicate $z > 6 \wedge x > z$:
- ▶ For each interaction above (3) which introduces a variable appearing in $z > 6 \wedge x > z$,
- ▶ add $z > 6 \wedge x > z$ in the predicate and quantify the variables accordingly:
 - ▶ \forall : the variables that A doesn't know and are introduced *before* (\rightarrow none)
 - ▶ \exists : the variables that are introduced *later* ($\rightarrow z$)

Lifting: Example

$$A \rightarrow B : \{x \mid x < 10 \wedge \exists z. x > z > 6\}. \quad (1)$$

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- ▶ For each interaction above (3) which introduces a variable appearing in $z > 6 \wedge x > z$,
- ▶ add $z > 6 \wedge x > z$ in the predicate and quantify the variables accordingly:
 - ▶ \forall : the variables that A doesn't know and are introduced *before* (\rightarrow none)
 - ▶ \exists : the variables that are introduced *later* ($\rightarrow z$)
- ▶ ... and check that the updated predicate is *satisfiable* (otherwise the algorithm is not applicable)
- ▶ ... repeat the algorithm.

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A Methodology

1. An architect designs a choreography
2. The architect is notified if there are any HS or TS problems
3. Using one of the 3 algorithms, solutions or hints of solution may be offered
4. The architect chooses which problem should be solved first
5. Steps 2 to 4 are repeated until all the problems have been solved

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Conclusions and Future Work

- ▶ Properties
 - ▶ Branching and recursion
 - ▶ Correctness
 - ▶ TS/HS preservation
 - ▶ Underlying type preservation
- ▶ Relation between algorithms
 - ▶ Strengthening vs Propagation
 - ▶ Strengthening vs Lifting
- ▶ Implementation

Thanks!

Any Questions?

Assertion Tree

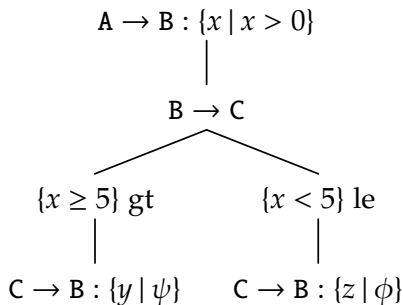
We often consider Global Assertions as trees, e.g. the global assertion

$$A \rightarrow B : \{x \mid x > 0\}.$$

$$B \rightarrow C : \{x \geq 5\} \text{ gt} : C \rightarrow B : \{y \mid \psi\}$$

$$\{x < 5\} \text{ le} : C \rightarrow B : \{z \mid \phi\}$$

can be represented by its parsing tree (called *Assertion Tree*)



In the following, \mathcal{G} ranges over global assertions, T over assertion trees, n over nodes, and $n \in T$ denotes that n is a node of T .

Some Definitions

We will use the following functions:

- ▶ $snd_T(n)$ returns the sender/selector for $n \in T$
- ▶ $rcv_T(n)$ returns the receiver for $n \in T$
- ▶ $var_T(n)$ returns the set of variables introduced at node $n \in T$
- ▶ $cst_T(n)$ returns the predicate of $n \in T$
- ▶ $PRED_T(n)$ returns the conjunction of all the predicates on the path from the root of T until the parent of n .

Strengthening: In General

- ▶ Let $n \in T$, where HS is violated for the variable v and let $\psi = cst_T(n)$.
- ▶ *Strengthening* consists in trying to find a variable v' such that
 - ▶ v' is known to $snd_T(n)$, and
 - ▶ $PRED_T(n) \wedge \psi[v'/v] \Rightarrow \psi$ holds.
- ▶ If such v' can be found, then *Strengthening* returns a new tree T' , obtained from T by replacing ψ by $\psi[v'/v]$ in n .

Propagation: In General

- ▶ Let $n \in T$ a node where HS is violated for the variable v .
- ▶ Propagation consists in finding a *chain* of (ordered) *interaction* nodes $n_1 \dots n_t$ such that:
 - ▶ $snd_T(n_1)$ knows v ,
 - ▶ $rcv_T(n_t) = rcv_T(n)$, and
 - ▶ for $1 \leq i < t$, $rcv_T(n_i) = snd_T(n_{i+1})$

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 - ▶ $snd_T(n_1)$ knows v ,
 - ▶ $rcv_T(n_t) = rcv_T(n)$, and
 - ▶ for $1 \leq i < t$, $rcv_T(n_i) = snd_T(n_{i+1})$
- ▶ and returning a new tree T' , updated from T such that
 - ▶ $cst_{T'}(n_1) = cst_T(n_1) \wedge (u_1 = v)$
 - ▶ for $2 \leq i < t$, $cst_{T'}(n_i) = cst_T(n_i) \wedge (u_i = u_{i-1})$
 - ▶ $cst_{T'}(n_t) = cst_T(n_t)[u_{t-1}/u]$
 - ▶ all the other nodes of T remain unchanged.

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- ▶ Propagation consists in finding a *chain* of (ordered) *interaction* nodes $n_1 \dots n_t$ such that:
 - ▶ $snd_T(n_1)$ knows v ,
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where each u_i variable is fresh and added in the corresponding node.